1501. Draw a force diagram for the first two parts. The answer to part (c) can be written down without calculation and without reference to the first two parts.

1502. The quartic $y = x(x-1)(x-2)^2$ is the minimal example of such a polynomial.

1503. By thinking about the $\pm$ in the quadratic formula, find the other root. Then use the factor theorem. Multiply out a factorised version of the equation.

——— Alternative Method ———

Equate ϕ to a copy of the quadratic formula with $a = 1$, and compare rational and irrational terms.

1504. In each case, consider the list of possible outcomes. Only one of the three is distributed binomially.

1505. (a) Write $x^2 + 4$ as $x(x+1) - (x+1) + 5$, then split the fraction up.
(b) Consider the behaviour as $x \to \infty$.

1506. To find the term, solve $u_n = 0$.

1507. Statement (a) is false. Find a counterexample.

1508. Solve the inequality $Y^2 > 4Y$ algebraically before considering the normal distribution.

1509. Split the exponential up using an index law.

1510. Set up *suvat* vertically and horizontally, ignoring the c and using x and y for vertical displacement. Eliminate t to find a Cartesian equation, and then translate the curve to take account of the starting point.

1511. This is an arithmetic series.

1512. Square both sides of the second statement.

1513. The roles of x and y have been reversed.

1514. This is a geometric progression. Use the standard formula for the nth term and then manipulate the algebra with index laws.

1515. Show that the points are collinear.

1516. (a) The (spatial) period is the wavelength.
(b) Consider the transformation associated with replacement of x by $3x$.
(c) Find $\text{lcm}(\text{period } 1, \text{period } 2)$.
(d) Again, find $\text{lcm}(\text{period } 1, \text{period } 2)$.

1517. At an x intercept, $f(x) = 0$.

1518. The variables in (a) and (c) are binomial.

1519. Using the quadratic formula, or by differentiation, find the equation of the line of symmetry of the graph.

1520. Multiply the brackets out first, then integrate.

1521. Use the chain rule to find the tangent line.

1522. (a) Since the second derivative is zero, the graph is a straight line (it has zero curvature).
(b) Use the result from part (a).

1523. Both are measures of spread, but only one takes account of all of the data.

1524. Set the first derivative to zero to find SPs, then evaluate the second derivative to classify them as minima (or use the shape of the curve).

1525. Consider the line of action of the weight when the cylinder is on the point of toppling: it must pass through the lowest point in contact with the slope.

1526. Substitute $(k, 0)$ and solve.

1527. This is a quadratic in $x^{\frac{5}{6}}$.

1528. (a) Find the accelerations using $F = ma$, and then set up *suvat* equations for the displacement of each ball.
(b) Set the result from (a) equal to 100.
(c) Find the modelled speed of the balls once they are 100 metres apart.

1529. Solve for b using the mean, then for a using the standard deviation.

1530. Express the curves in generic algebraic form, and set up an equation to solve for intersections.

1531. For both parts, visualise the equations as a pair of parallel lines.

1532. Find the position vectors of M and N using means, then find the mean of them.

1533. Consider the x axis intercepts.

1534. Find the radius and centre of the circle.

1535. You can list these by cases, according to whether there are $0, 1, 2, 3, 4, 5$ blue edges.

1536. Take out a factor of $(3x-1)^2$.

1537. First, solve the boundary equations $x^2 = 20$ and $x^3 = 50$. Then list the constituent sets.

1538. Resolve the forces along their line of symmetry.

1539. This is true; explain why.

1540. To calculate the distance to a circle, calculate the distance to its centre. You'll also need to compare these to the radius.

1541. Expand and equate rational and irrational parts. Solve the resulting pair of simultaneous equations.

1542. Differentiate to find the equation of the normal at the origin. Then solve this simultaneously with the curve to find the intersections, and set up a definite integral for one of the regions.

1543. Give a specific counterexample.

1544. For both parts, rearrange to the form $a^p = a^q$. The indices can then be equated, giving $p = q$.

1545. A root satisfies $\text{f}(x) = 0$, whereas a fixed point satisfies $\text{f}(x) = x$.

1546. (a) Use the product rule $\dfrac{d}{dt}(xy) = \dfrac{dx}{dt}y + x\dfrac{dy}{dt}$.
   (b) Substitute the initial conditions into $\dfrac{dA}{dt}$.
   (c) Set $\dfrac{dA}{dt} = 0$ and solve.

1547. Multiply up by the denominators, taking care with the minus signs.

1548. This is a key map in the *Four Colour Theorem*: it requires four different colours to colour it.

1549. (a) Use the formula $y - y_1 = m(x - x_1)$, with V playing the role of y and T the role of x.
   (b) Integrate between $T = 0$ and $T = t$. Note carefully the difference in meaning between v (final velocity) and V (variable velocity), and between t (final time) and T (variable time).

1550. Increasing/decreasing refers to the sign of the first derivative, i.e. the gradient. Convex/concave refers to the sign of the second derivative, i.e. the rate of change of the gradient or curvature.

1551. Use log laws.

1552. (a) By setting the derivative to zero, show that both SPs are the same side of the line $y = 1$.
   (b) Factorise to cubic to sketch it. Consider the multiplicity of its roots.
   (c) Reflection in $y = 1$ is the same as reflection in $y = 0$ followed by translation by vector $2\mathbf{j}$.

1553. (a) The integrand is $\pi\big(r^2 - x^2\big)$. Consider the equation of a circle $x^2 + y^2 = r^2$.
   (b) Carry out the integral. Note that x is variable, while r is constant.

1554. The signed area between the parabola and the x axis has a maximum (as opposed to a minimum), so it must be a negative parabola. Consider the contribution of the sections below the x axis.

1555. This is already factorised, so can be treated as two separate equations. There are four roots.

1556. Think of a counterexample to the first sentence: the initial claim is not always true. And, even when it is true, it has nothing to do with NIII!

1557. Set up an equation with the output equal to y, then rearrange to make x the subject.

1558. (a) Set to 0 and multiply by $\sqrt{x}$. Then spot an obvious root and factorise. You might want to use a polynomial solver to help you.
   (b) Integrate definitely between the intercepts.

1559. This is true. Consider the boundary scenario in which the LHS, say, is an equality.

1560. Take natural logs of both sides, and use log rules.

1561. (a) The logic you need is $\text{f}(*) = 2 \implies * = p$.
   (b) Solve two equations.

1562. The inequalities are satisfied by the points above and to the right of the relevant boundary line. Your counterexamples should be specific (x, y) points.

1563. This isn't a probability question, in fact. It's a question of pure algebra. Use the discriminant.

1564. Start with $(x^2+1)^2$ to match the x^4 term, then consider the term in x^2.

1565. (a) Set up simultaneous equations in u and a using the period $t \in [0, 2]$ and the period $t \in [0, 4]$.
   (b) Use *suvat*.
   (c) Find the displacement over the 10 seconds.

1566. Use -15 to find a, then expand to find b.

1567. (a) Use the standard result that, if $X \sim N(\mu, \sigma^2)$, then the mean of a random sample of size n has distribution $\bar{X} \sim N(\mu, \sigma^2/n)$.
(b) Use the normal facility on a calculator.

1568. Consider this graph as a transformation of $xy = 1$.

1569. Carry out the integrals on each side, gathering the two constants of integration into one $+c$ on the LHS. Then exponentiate both sides. Use an index rule to simplify. At the end, rename e^c as A.

1570. Integrate a quadratic, and you get...?

1571. Find the coordinates of the vertices, or else use a symmetry argument.

1572. Set up simultaneous equations, using the steps $x_1 \mapsto x_2$ and $x_2 \mapsto x_3$.

1573. Resolve perpendicular to the acceleration to find the angle, then resolve parallel to the acceleration. Alternatively, use a triangle of forces.

1574. Statements (b) and (c) are the false ones.

1575. Consider the range (set of outputs) of sine. You don't need to worry about the inputs here.

1576. (a) Use the cumulative distribution facility on a calculator.
(b) Consider the skewness of the distribution and the size of the sample.

1577. Sketching the relevant boundary graphs. They are tangent to one another at $(1, 1)$.

1578. To factorise this, you can factorise the LHS of

$$39867x^2 - 93574x - 57893 = 0.$$

Use the quadratic formula and the factor theorem.

1579. Differentiate to find the gradient in terms of p. Then substitute this gradient and the point (p, p^2) into the formula $y - y_1 = m(x - x_1)$.

1580. Find the position at time t of each, taking $t = 0$ to be the time at which the second projectile is dropped. The first projectile, then, will have fallen for $t + 1$ seconds at time t.

1581. Logarithms are undefined for negative inputs.

1582. (a) The vectors share no components. Visualise this in a 2D context if you need to.
(b) Use Pythagoras.

1583. The possibility space is getting a bit big to draw it explicitly. Visualise it, and count outcomes.

1584. Differentiate by the chain rule, then use a Pythagorean trig identity.

1585. A proof by contradiction begins with the *opposite* of what one is trying to prove.

1586. Express A in terms of a variable side length x and the constant P. Then differentiate with respect to x and set the derivative to zero for optimisation.

1587. (a) Substitute the proposed solution.
(b) Spot the value of k which makes the two sides identical. Or substitute $x = 0$.

1588. Place the vertices of T at the midpoints of the sides of S. Then use trig or Pythagoras to find the length scale factor between the triangles.

1589. In (a),(b),(c), simply transform the lower bound 1. Only in (d) does the form of the range change.

1590. Consider the fact that a string can go slack.

1591. Differentiate with respect to y to find dx/dy. Then reciprocate both sides of that equation. Evaluate at $y = -2$ and use $y - y_1 = m(x - x_1)$.

1592. (a) Use the fact that probabilities are positive and sum to 1.
(b) Restrict the possibility space.

1593. Multiply both sides by $x^4 - x^2$, and then equate coefficients.

1594. Use Δ or factorise to show that there is a double root in the equation for intersections.

1595. Use the second Pythagorean trig identity.

1596. Consider the fact that f and g may be the same quadratic function.

1597. Equate the differences and solve a quadratic.

1598. Squaring $\sqrt{y} = x^2 - 1$ gives $y = (x^2 - 1)^2$. So, all points on the given curve appear on your curve. The reverse is not true, however.

1599. Visualise the car floating in space.

1600. Explaining why something is not true is often most easily done by producing a clear counterexample. Consider a sample from a bimodal distribution.

——— End of 16th Hundred ———